

Boosting some type-D metrics[★]

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Abstract

We are presenting a general solution to the classical Einstein–Maxwell–dilaton–axion equations starting from a metric of type-D. Namely, this stringy solution is the result of a transformation on a general vacuum type-D solution to the Einstein’s equations which was studied in detail some years ago.

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1. Introduction

There has been a good deal of work on several topics around the low energy effective action of string theory. In particular, the analysis of the equations of motion of the dilaton and/or axion – graviton system has given rise to solutions that exhibit some new properties of the black holes (or black p-branes) of this string gravity [1-6]. One of the first important analyses of the classical dilaton–graviton system was reported in [1], which shows that the physical properties of the black hole solutions vary depending on the value of the coupling constant of the dilaton field (e.g. black hole thermodynamics, stability of inner horizons and so on).

Similar work for the dilaton – gravity system has been presented in [2] and extended in [3] to higher dimensions with solutions associated with the so-called black strings and black p-branes. An interesting approach was adopted in [4] while studying the axion-gravity system, using an embedding–boosting–duality sequel of steps enabling generation of an isometry, a conserved momentum in the translation invariant direction and finally by duality [7] an axion charge that can be associated with the conserved momentum.

Later, an important step in the construction of dyonic solutions of the string equations was given in [8], in which the exact $SL(2, R)$ symmetry of the equations of motion of the low energy effective string action in four dimensions was used to construct a dual solution with electric and magnetic charge.

A systematic method for studying the space of solutions of the equations of motion of the LEEA of the heterotic string is given in [15] (see also refs. therein). The main ingredients that Sen uses are a realisation of the $O(d, d + p)$ duality symmetry [9, 11, 12, 13] and elements of the subgroup $O(d - 1, 1) \times O(d + p - 1, 1) \subset O(d, d + p)$ to generate the new solutions. This kind of symmetry arises in the process of compactification. For example, the 10-dimensional $N = 1$ supergravity with p abelian vector supermultiplets (in addition to the supergravity multiplet) is dimensionally reduced to $10 - d$ dimensions with a global $O(d, d + p)$ symmetry [14, see refs therein].

The $O(7, 23)$ group has been used to generate charged solutions [16 - 20] from

a neutral solution with time independent field configurations including the moduli fields. The $O(8, 24)$ group of symmetries of string theories was studied and used to generate a general charged solution [18, 20]. The case with $SO(2, 3)$ as the symmetry group has been studied in [21] in which the moduli fields are not considered and an equivalent action that reproduces the equations of motion of the LEEA has been used. There is a further reduction to two-dimensions that leads to an infinite dimensional symmetry $O(\widehat{8, 24})$ [22, 23, see refs. therein]. All these symmetries can be seen as the extensions of the symmetries studied in the Einstein-Maxwell theory such as Ehlers-Harrison transformations, the Geroch group and the Kinnersley-Chitre symmetries [24, 25, 26].

A realisation of the $O(6, 22)$ symmetry is given by the LEEA in four dimensions

$$S = \int d^4x \sqrt{-g} (R(g) - \frac{1}{2}(\nabla\Phi)^2 - \frac{1}{12}e^{-2\Phi}H^2 - \frac{1}{8}e^{-\Phi}F^2 + \dots) \quad (1)$$

where Φ is the dilaton field, $F_{\mu\nu}$ is the Maxwell field associated with a $U(1) \subset E_8 \times E_8$ gauge field, $g_{\mu\nu}$ is the Einstein metric that is related to the string metric as follows $g_{\mu\nu} = e^{-\Phi}G_{\mu\nu}$. The three-form with only the Chern-Simons term for the $U(1)$ gauge field is given by

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{4}A_\mu F_{\nu\rho} + \text{cyclic permutations} \quad (2)$$

where B is the antisymmetric tensor gauge field.

Considering only one copy of the $U(1)$ gauge field, the background field configuration can be contained in the 9×9 matrix M as follows [16]

$$M = \begin{pmatrix} K_-^T G^{-1} K_- & K_-^T G^{-1} K_+ & -K_-^T G^{-1} A \\ K_+^T G^{-1} K_- & K_+^T G^{-1} K_+ & -K_+^T G^{-1} A \\ -A^T G^{-1} K_- & -A^T G^{-1} K_+ & A^T G^{-1} A \end{pmatrix} \quad (3)$$

where

$$K_\pm = -B - G - \frac{1}{4}AA^T \pm \eta_4$$

with $\eta_4 = \text{diag}(+, +, +, -)$.

A new field configuration can be obtained from the following transformation [16]

$$M'(G', B', A') = \Omega M \Omega^T \quad (4)$$

and the dilaton field transforms under the duality rules [7]

$$\Phi' = \Phi - \frac{1}{2} \log\left(\frac{\text{Det}G}{\text{Det}G'}\right) \quad (5)$$

with Ω given by

$$\Omega = \begin{pmatrix} I_7 & 0 & 0 \\ 0 & \cosh \alpha & \sinh \alpha \\ 0 & \sinh \alpha & \cosh \alpha \end{pmatrix} \quad (6)$$

where I_7 is the identity matrix 7×7 . Then, one can generate (in general) inequivalent backgrounds which are time independent [17] as this subgroup leaves invariant the equations of motion. We can start from a given solution of the pure Einstein's equation (i.e. $A_\mu = B_{\mu\nu} = \Phi = 0$) and construct a solution of the full string theory equations of motion.

In addition to the $O(d, d+p)$ symmetries, there is also an exact $SL(2, R)$ symmetry of the equations of motion of the LEEA of the heterotic string in four dimensions that enables construction of dyonic solutions starting from one configuration with either an electric or magnetic charge. The equations of motion of the action (1) are invariant under a $SL(2, R)$ duality symmetry of the type [8, 17]

$$F_- \rightarrow -\bar{\lambda} F_-, \quad F_+ \rightarrow -\lambda F_+ \quad (7)$$

$$\lambda \rightarrow \lambda + c, \quad \lambda \rightarrow -1/\lambda \quad (8)$$

with

$$F_\pm^{\mu\nu} = F^{\mu\nu} \pm i \tilde{F}^{\mu\nu}$$

$$\lambda = \Psi + ie^{-\Phi} \tag{9}$$

the imaginary part of F is

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} F_{\rho\sigma}$$

The real part of the modular parameter λ is given by the axion field and its imaginary part is given by the dilaton field. This can be seen as one of the manifestations of the S-duality symmetry in the LEEA [29, 19]. The $SL(2, \mathbb{Z})$ symmetry has also appeared in lattice gauge theories with a theta-term [10, 11], where the models without θ -term are self-dual with respect to inversion of the coupling constant and with a θ -term this duality may be extended to an invariance $D : \zeta \rightarrow -1/\zeta, T : \zeta \rightarrow \zeta + 1$ where ζ is a complex coupling constant. This kind of duality resembles the transformation (8) of the coupling (9) in the case of the heterotic LEEA [8].

In section 2, we shall briefly review the most general type D metric and construct a solution to the heterotic string equations starting from a generic type D metric which is a solution to the Einstein equations. The method uses an $O(1, 1)$ transformation and only one copy of a $U(1)$ gauge field. Next, we shall use the $SL(2, \mathbb{R})$ transform to generate the dual solution. We should like to investigate the symmetric role played by the parameters and the coordinates on which the field configuration depends. Finally, we shall present the string version of the C-metric [33].

One should note that the $SL(2, \mathbb{R})$ transformation together with the $O(7, 23)$ generate the $O(8, 24)$ group. Then, using an element of $O(8, 24)$ with one single transformation, it is possible to generate a 59 parameter solution provided that one is transforming a time independent configuration [19, 20], then giving rise to solutions that contain the previous solutions discussed in [2, 5, 16, 27, 28].

2. Type-D solutions yesterday and today

In solving the Einstein–Maxwell equations the most general type-D metric has been given in [31] and reads as follows

$$ds^2 = \frac{1}{(1-pq)^2} \left[\frac{p^2+q^2}{P} dp^2 + \frac{P}{p^2+q^2} (d\tau + q^2 d\sigma)^2 + \frac{p^2+q^2}{Q} dq^2 - \frac{Q}{p^2+q^2} (d\tau - p^2 d\sigma)^2 \right] \quad (10)$$

These space-times are described by some real coordinates $x^\mu = (p, q, \sigma, \tau)$ and with signature $(+, +, +, -)$. $P(p)$ and $Q(q)$ are quartic polynomials

$$P(p) = (-\lambda/6 - g_0^2 + \gamma) + 2np - \epsilon p^2 + 2mp^3 + (-\lambda/6 - e_0^2 - \gamma)p^4 \quad (11)$$

and

$$Q(q) = -P(-q) \quad (12)$$

The parameters m, n, e_0, g_0 (mass, NUT parameter, electric and magnetic charge) are the dynamical parameters that enter into the electromagnetic field and the curvature. The parameters γ and ϵ are kinematical parameters related to the rotation and to some constant acceleration of a particle with mass m . The parameter λ is the cosmological constant.

The electromagnetic field is given by

$$f_+ \equiv (f_{\mu\nu} + i\tilde{f}_{\mu\nu})dx^\mu dx^\nu = -d\left(\frac{e_0 + ig_0}{q + ip}(d\tau - ipq d\sigma)\right) \quad (13)$$

with $\tilde{f}^{\mu\nu} = \frac{1}{2}(-g)^{-1/2}\epsilon^{\mu\nu\rho\sigma}f_{\rho\sigma}$. The complex invariant $\mathcal{F} = \frac{1}{2}(f_{+\mu\nu})^2 = f_{\mu\nu}f^{\mu\nu} + if_{\mu\nu}\tilde{f}^{\mu\nu}$ is

$$\mathcal{F} = -\frac{1}{2}(e_0 + ig_0)^2 \left(\frac{1-pq}{q+ip}\right)^4 \quad (14)$$

and the only nonvanishing component of the Weyl tensor is

$$C^{(3)} = -2\left(\frac{1-pq}{q+ip}\right)^3 [(m+in) - (e_0^2 + g_0^2)\frac{1+pq}{q-ip}] \quad (15)$$

It would be interesting to see the extension of this general metric in the context of

string theory. For this, we will first use a generic type-D neutral solution to generate a charged solution to the equations of motion of the action (1), that includes the dilaton and axion fields.

2.1 Type D metrics without conformal factor and their stringy version

From (10) one obtains a class of solutions where the conformal factor has been removed [30]. That is equivalent to eliminate the parameter of acceleration

$$ds^2 = \frac{p^2 + q^2}{P} dp^2 + \frac{P}{p^2 + q^2} (d\tau + q^2 d\sigma)^2 + \frac{p^2 + q^2}{Q} dq^2 - \frac{Q}{p^2 + q^2} (d\tau - p^2 d\sigma)^2 \quad (16)$$

with the structural functions given by

$$P(p) = \gamma - g_0^2 + 2np - \epsilon p^2 - (\lambda/3)p^4 \quad (17)$$

$$Q(q) = \gamma + e_0^2 - 2mq + \epsilon q^2 - (\lambda/3)q^4 \quad (18)$$

This solution contains six continuous and a discrete parameters. From which ϵ can take the values $\pm 1, 0$. The complex invariant is given by

$$\mathcal{F} = -\frac{1}{2}(e_0 + ig_0)^2 \frac{1}{(q + ip)^4} \quad (19)$$

The only nonvanishing component of the Weyl tensor

$$C^{(3)} = \frac{-2}{(q + ip)^3} [(m + in) - (e_0^2 + g_0^2) \frac{1}{q - ip}] \quad (20)$$

We consider the neutral solution (16) - (18) with $e_0 = g_0 = 0$ to generate a charged solution by using an $O(1, 1)$ transformation given by Ω in (4) - (6). From the properties of the starting metric we should obtain a stringy metric with all the known parameters except the acceleration one. The new charges (electric and magnetic) will be consistent with the presence of the dilaton and axion fields.

The new metric ds'^2 , the electromagnetic gauge field A_μ and the antisymmetric field $B_{\mu\nu}$ are obtained according to (4). The dilaton field is obtained from (5), thus, the resulting new Einstein metric given by $ds_E^2 = e^{-\Phi} ds'^2$ is

$$ds_E^2 = \frac{D}{P} dp^2 + \frac{P}{D} (d\tau + (b_1 q^2 + b_0 Q) d\sigma)^2 + \frac{D}{Q} dq^2 - \frac{Q}{D} (d\tau - (b_1 p^2 - b_0 P) d\sigma)^2 \quad (21)$$

with

$$D = b_1(p^2 + q^2) - b_0(P - Q)$$

and

$$b_0 = \frac{(1 - \cosh \alpha)}{2}; \quad b_1 = \frac{(1 + \cosh \alpha)}{2}$$

The scalar dilaton field, according to (5) is given by

$$e^\Phi = \frac{(p^2 + q^2)}{D} \quad (22)$$

The gauge fields are

$$A_\tau = (p^2 + q^2 + P - Q) \frac{\sinh \alpha}{D} \quad (23)$$

$$A_\sigma = (Pq^2 + p^2Q) \frac{\sinh \alpha}{D} \quad (24)$$

and the antisymmetric tensor is

$$B_{\tau\sigma} = \frac{b_0}{D} (Pq^2 + p^2Q) \quad (25)$$

The axion field associated with antisymmetric field can be obtained from (2) and the equation for $H_{\mu\nu\rho}$, that gives rise to the expression $H^{\mu\nu\rho} = -\frac{1}{\sqrt{-g}} e^{2\Phi} \epsilon^{\mu\nu\rho\sigma} \partial_\sigma \Psi$.

After we have fixed the parameters $\epsilon = 1$ and $\lambda = 0$ in the structural functions P and Q , one obtains the scalar axion field

$$\Psi = 2b_0 \frac{(nq - mp)}{p^2 + q^2} \quad (26)$$

From the solution presented above with $\gamma = 0$ (21) - (25), one can derive the solution given in [32], which has been obtained by direct integration of the equations of motion.

The asymptotic expansion of the metric, dilaton and electromagnetic potential allows to interpret new parameters corresponding to the mass, an scaled NUT-parameter, electric and magnetic charges, and dilaton and axion charges as

$$M = b_1 m; \quad e' = 2m \sinh \alpha; \quad d_0 = b_0 m$$

$$N = b_1 n; \quad g' = 2n \sinh \alpha; \quad a_0 = b_0 n$$

where we have used the coordinate transformation

$$q \rightarrow r; \quad p \rightarrow a \cos \theta - \frac{2b_0 m^2}{n}$$

leading to expansions such as

$$-g_{00} \sim 1 - \frac{2M}{r} - \frac{2Na \cos \theta}{r^2} + \dots$$

$$A_0 \sim \frac{e'}{r} + \frac{g'a \cos \theta}{r^2} + \dots$$

$$e^\Phi \sim 1 + \frac{2d_0}{r} + \frac{2d_0 a \cos \theta}{r^2} + \dots$$

$$\Psi \sim \frac{2a_0}{r} + \dots$$

We can also rewrite (21) as

$$\begin{aligned}
ds_E^2 = & \frac{D}{P}dp^2 + \frac{P}{D}(d\tau + Q_\Phi d\sigma)^2 \\
& + \frac{D}{Q}dq^2 - \frac{Q}{D}(d\tau - P_\Psi d\sigma)^2
\end{aligned} \tag{27}$$

where

$$Q_\Phi = b_0\gamma - 2d_0q + q^2; \quad P_\Psi = -b_0\gamma - 2a_0p + p^2$$

We have then obtained a general solution to the heterotic string equations of the action (1), that includes dilaton and axion fields. This solution contains all the known dynamical and kinematical parameters except the acceleration parameter. This parameter can presumably be included by adding a conformal factor $\frac{1}{(1-pq)^2}$ to (16) then having (10) as the starting metric to generate a more general solution.

Similarly as with their counterparts in the Einstein-Maxwell theory the electromagnetic field components can be aligned with the directions

$$e_3(e_4) = \frac{1}{\sqrt{2}}\left\{\left(\frac{D}{Q}\right)^{1/2}dq \pm \left(\frac{Q}{D}\right)^{1/2}(d\tau - P_\Psi d\sigma)\right\}$$

In this case, as it has been presented in [32] the values of the Weyl coefficients indicate that these kinds of gravitational fields are algebraically general as opposed to the type-D characteristic of the Einstein-Maxwell solution (16) - (20) [30].

2.2 Dyonic solutions from type-D metrics without conformal factor

In the solution previously presented the NUT parameter allows to generate a dyonic solution with electric and magnetic charge. One can also generate a new dyonic solution by using the fact that the equations of motion of the action (1) are invariant under an $SL(2, R)$ transformation [11, 17, 28]. Then as for the case of electromagnetic field coupled to gravity their equations of motion are invariant under duality transformations in which the electric and magnetic fields are continuously rotated into one another allowing solutions that carry both electric and magnetic charge.

The transformations (7) and (8) give rise to a new electromagnetic field

$$\sqrt{1+c^2}\hat{f}_{\mu\nu} = -(\Psi+c)f_{\mu\nu} + e^{-\Phi}\tilde{f}_{\mu\nu} \quad (28)$$

and new axion and dilaton fields [17]

$$\hat{\Psi} = -\frac{(1+c^2)(\Psi+c)}{(\Phi+c)^2 + e^{-2\Phi}} \quad (29)$$

$$e^{-\hat{\Phi}} = \frac{(1+c^2)e^{-\Phi}}{(\Phi+c)^2 + e^{-2\Phi}} \quad (30)$$

keeping the Einstein metric (27) invariant. In the case we are dealing with, then

$$\hat{\Psi} = -\frac{p^2 - 2d_0p + q^2 + 2a_0q}{(p-d_0-a_0)^2 + (q-d_0+a_0)^2}$$

$$e^{-\hat{\Phi}} = \frac{p^2 - 2a_0p + q^2 - 2d_0q}{(p-d_0-a_0)^2 + (q-d_0+a_0)^2}$$

where we have set the parameter $c = 1$. They have very similar expressions with a new dilaton charge a_0 and an axion charge $-d_0$. From the electromagnetic fields (28), whose expressions are more involved, one can obtain a new electric charge $Q_E = 2(m+n)\sinh\alpha$ and a magnetic charge $Q_M = 2(m-n)\sinh\alpha$, provided that $m \neq n$.

3.3 The C-metric and its stringy version

We obtained the general solution that includes the solutions previously studied [16, 27, 28, 21, 32]. Now it would be interesting to include the conformal factor as in the case of the Einstein-Maxwell theory. One of the simplest cases with this conformal factor is the so-called C-metric. With a contraction procedure the metric (10) can be reduced to a Kinnersley-Walker type of metric [33]

$$ds^2 = \frac{1}{(p+q)^2} \left[\frac{1}{P(p)} dp^2 + \frac{1}{Q(q)} dq^2 + P(p) d\sigma^2 - Q(q) d\tau^2 \right] \quad (31)$$

with

$$P(p) = \left(\gamma - \frac{\lambda}{6}\right) + 2np - \epsilon p^2 + 2mp^3 - (e^2 + g^2)p^4 \quad (32)$$

$$Q(q) = -\left(\gamma + \frac{\lambda}{6}\right) + 2nq + \epsilon q^2 + 2mq^3 + (e^2 + g^2)q^4 \quad (33)$$

with the electromagnetic field as follows

$$f_+ = d((e + ig)(qd\tau + ipd\sigma)) \quad (34)$$

and the invariant

$$\mathcal{F} = -\frac{1}{2}(e + ig)^2(p + q)^4 \quad (35)$$

The only nonvanishing component of the Weyl tensor is

$$C^{(3)} = 2m(p + q)^3 - 2(e^2 + g^2)(p + q)^3(p - q) \quad (36)$$

Let us consider the case with $\lambda = n = e^2 + g^2 = 0$, which can be taken to the form

$$ds^2 = \frac{1}{A^2(x+y)^2} \left[\frac{1}{F(y)} dy^2 + \frac{1}{G(x)} dx^2 + G(x) dz^2 - F(y) dt^2 \right] \quad (37)$$

where

$$G(x) = 1 - x^2 - 2mA x^3 \quad (38)$$

$$F(y) = -1 + y^2 - 2mA y^3 \quad (39)$$

This space-time has been interpreted as the space-time of two particles with uniform

acceleration A [33].

The idea now is to construct a stringy version of this space-time by using the technique described above. After this $O(1,1)$ transformation the charged solution obtained is given by

$$ds'^2 = \frac{1}{A^2(x+y)^2} \left[\frac{1}{F(y)} dy^2 + \frac{1}{G(x)} dx^2 + G(x) dz^2 - \frac{A^4(x+y)^4}{D^2} F(y) dt^2 \right] \quad (40)$$

where

$$D = b_0 F(y) + b_1 A^2(x+y)^2 \quad (41)$$

In this new configuration the dilaton field given by (5) is

$$e^\Phi = \frac{A^2(x+y)^2}{D} \quad (42)$$

Now, we pass on the expression of the new stringy metric as an Einstein metric $ds_E = e^{-\Phi} ds'^2$, thus

$$ds_E^2 = \frac{1}{A^2(x+y)^2} \left[\frac{1}{e^\Phi F(y)} dy^2 + e^{-\Phi} \left(\frac{1}{G(x)} dx^2 + G(x) dz^2 \right) - e^\Phi F(y) dt^2 \right] \quad (43)$$

The nonvanishing electromagnetic potential is given by

$$A_t = \frac{A^2(x+y)^2 - F(y)}{D} \sinh \alpha \quad (44)$$

and the antisymmetric field remains zero. The properties of this space-time will be presented elsewhere.

3. Summary

In this work we have presented a general solution to the equations of motion of the heterotic string using duality transformations. Since the transformation that we have used [16, 17] only modifies the dynamical parameters of the starting metric, namely the mass, the NUT-parameter, the electric and magnetic charges, leaving the kinetical parameters unmodified, then it is possible to extend the known seven-parameter solution of the Einstein-Maxwell theory [31] to a new field configuration consistent with the presence of the dilaton and axion fields. In this case the new metric can also contain the parameter of acceleration. We should notice that the electromagnetic field can be aligned with some null directions alike the case of the Einstein-Maxwell theory.

We have given a solution that contains the previous constructions [16, 27, 28, 21, 32] in which only some of the parameters were included. We have also given a particular solution that extends the so-called C-metric to the context of string theory. This kind of metric has been of interest in the study of pair-particle creation near an event horizon [34], as it can be interpreted as an space-time representing two particles with constant acceleration. This aspect will be explored further elsewhere.

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